Homework 6 Examples

Example 1

Use the following statements to prove the conclusion reached in the argument: "Students who go to class do well on the test." "Good students go to class." "Phillip is a good student." "Therefore, Phillip did well on the test."

Step 1: Create symbols to represent each idea in the statements. (0.5 points for clearly defining symbols)

```
Symbols:
	C(x) = x goes to class
	W(x) = x does well on the test
	G(x) = x is a good student.
```

Step 2: Create the premises by turning the statements into predicates using the symbols. (0.5 points for each premise per statement)

```
Premises:
	∀x(C(x) → W(x))
	∀x(G(x) → C(x))
	G(Phillip)
```

Step 3: Label the conclusion by turning the concluding statement into a predicate. (0.5 points for correctly labelling the conclusion)

```
Conclusion:
	W(Phillip)
```

Step 4: Give the proof leading from the premises to the conclusion. (roughly 0.5 points per step of proof; 0.25 for the actual content of the step being correct, 0.25 for the correct label or rule of inference applied for the step; note that steps don't need to be in a specific order, but a good rule of thumb is to put premises as close to where they are used in rules of inference)

```
Proof:
   1. ∀x(G(x) → C(x))          Premise
   2. G(Phillip) → C(Phillip)   Instantiation 1
   3. G(Phillip)                Premise
   4. C(Phillip)                Modus Ponens 2, 3
   5. ∀x(C(x) → W(x))          Premise
   6. C(Phillip) → W(Phillip)   Instantiation 5
   7. W(Phillip)                Modus Ponens 4, 6
```

Example 2

What conclusions can you reach using the following statements? "When you like chocolate ice cream and you are lactose intolerant, you get a stomach ache." "Ephraim is lactose intolerant." "Rebecca does not have a stomach ache."

Step 1: Create symbols to represent each idea in the statements. (0.5 points for clearly defining symbols)

Symbols:

```
C(x) = x likes chocolate ice cream
L(x) = x is lactose intolerant
S(x) = x has a stomach ache
```

Step 2: Create the premises by turning the statements into predicates using the symbols. (0.5 points for each premise per statement)

Premises:

```
∀x(C(x) ∧ L(x) → S(x))
L(Ephraim)
~S(Rebecca)
```

Step 3: Use logic to develop new valid conclusions. (roughly 0.5 points per step of logic; 0.25 for the actual content of the step being correct, 0.25 for the correct label or rule of inference applied for the step)

Logic:

1.	∀x(C(x) ∧ L(x) → S(x))	Premise
2.	C(Rebecca) ∧ L(Rebecca) → S(Rebecca)	Instantiation 1
3.	~S(Rebecca)	Premise
4.	~(C(Rebecca) ∧ L(Rebecca))	Modus Tollens 2, 3
5.	~C(Rebecca) ∨ ~L(Rebecca)	DeMorgan's 4

Step 4: Label the conclusion by turning the concluding statement into a predicate. (1.5 points for correctly labelling each possible conclusion)

Conclusions:

- Rebecca does not like chocolate ice cream or Rebecca is not lactose intolerant.

Example 3

Use resolution to answer the following datalog query:

```
Schemes:
  A(X)
  B(X)
  C(X,Y)
Facts:
  A('m')
  B('p')
Rules:
  C(X,Y) :- A(X), B(Y).
Queries:
  C('m',X)?
```

Step 1: Acknowledge that the symbols you will use are in the Schemes section. Not using these symbols will deduct points (0.5 points).

Step 2: Acknowledge that every fact is a premise. Include it in the list of premises. (0.5 points for each fact).

Step 3: Turn each rule into a premise by "AND"ing every predicate on the right side of the COLON_DASH, putting the newly ANDed predicates on the left side of an implication, and putting the head predicate of the rule on the right side of the implication. Finally, put that implication in a universal quantifier. (0.5 points for each correct rule).

```
Premises:
      A('m')
      B('p')
      ∀X∀Y(A(X) ∧ B(Y) → C(X,Y))
            rewritten for resolution as: ∀X∀Y(~A(X) ∨ ~B(Y) ∨ C(X,Y))
```

Step 4: Turn the query into a conclusion by putting the predicate in an existential quantifier. (0.5 points for a correct conclusion)

```
Conclusion:
      ∃X(C('m',X))
```

Step 5: Build a proof by contradiction with the conclusion using resolution and instantiation. (roughly 0.25 points for each correct step of the proof)

Proof:

1.	~∃X(C('m',X))	Proof by contradiction
2.	∀X(~C('m',X))	DeMorgan's 1
3.	~C('m','p')	Instantiation 2
4.	∀X∀Y(~A(X) ∨ ~B(Y) ∨ C(X,Y))	Premise
5.	~A('m') ∨ ~B('p') V C('m','p')	Instantiation 4
6.	~A('m') ∨ ~B('p')	Resolution 4, 5
7.	A('m')	Premise
8.	~B('p')	Resolution 6, 7
9.	B('p')	Premise
10.	FALSE	Resolution 8, 9